

# A Bayesian semiparametric model for semicontinuous data

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## Abstract

When the target variable exhibits a semicontinuous behaviour (i.e. a point mass in a single value and a continuous distribution elsewhere) parametric ‘two-part regression models’ have been extensively used and investigated. In this paper, a semiparametric Bayesian two-part regression model for dealing with such variables is proposed. The model allows a semiparametric expression for the two part of the model by using Dirichlet processes. A motivating example (in the ‘small area estimation’ framework) based on pseudo-real data on grapewine production in Tuscany, is used to evaluate the capabilities of the model. Results show a satisfactory performance of the suggested approach to model and predict semicontinuous data when parametric assumptions (distributional and/or relationship) are not reasonable.

**Keywords:** Dirichlet processes; Hierarchical Bayesian models; Small area estimation; Two-part models.

## 1 Introduction

In many field of applications, in particular in biomedical and economic studies, researchers encounter data that are either continuous on the positive line or zero. In literature, such data, in which the zeros are actual response outcomes and not proxies for negative or missing responses, are referred as semicontinuous and are usually handled by a non-standard two component mixture; whose terms are a degenerate distribution (a point mass at zero) and some standard distribution. This is realized by carrying out two regression models, one for the mixing proportion (usually logit or probit), the other

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for the mean of the standard distribution. The latter is a conditional regression model that depends on the nature of the data. Models of this type, are commonly called two-part models and have large use in econometrics (Duan *et al.*, 1983, among others) and for the analysis of longitudinal data in biomedical applications (Olsen and Shafer, 2001; Berk and Lachenbruch, 2002; Tooze *et al.*, 2002; Albert and Shen, 2005; Gosh and Albert, 2009). In this latter context, in order to account for both the heterogeneity among individuals and the possible correlation among subsequent observations on the same individual, a cluster-specific random effect is usually enclosed in both the parts of the model. More recently, the use of a two-part random effects model has been suggested also in the context of small area estimation (Pfeffermann *et al.*, 2008; Dreassi *et al.*, 2014; Chandra and Chambers, 2014).

In this paper, we propose a semiparametric Bayesian two-part model where the model for the mixing proportion is semiparametric and the model for the mean of the positive response outcomes is nonparametric. The first one is a binary regression model in which the commonly used parametric link function is replaced by a general function for which a Dirichlet process (DP hereafter), centered around a logistic distribution, is employed as prior distribution. In the second one, a DP mixture of Normals is specified for the joint distribution of the response and predictors. When model assumptions on conditional distributions are not acclaimed, this approach, reducing the need for parametric assumptions, brings down model specification errors.

Into a Bayesian paradigm, Bayesian nonparametric conditional density regression models, have been widely used: Escobar (1994) and Escobar and West (1995) discuss computational issues in DP mixture models where a parametric prior in a hierarchical model is replaced by the nonparametric DP model; Muller *et al.* (1996) and Dunson *et al.* (2007) face the problem of density regression using Bayesian semiparametric and nonparametric approaches. Fitted regression functions may be deduced as means of conditional predictive distributions. The use of the DPs offers great capabilities to furnish predictive distribution in a straight way; so, if the main goal of the statistical analysis is prediction, their use is very attractive. Another important characteristic of DP mixtures is to define clusters on data. These latter seem very appealing when structured data are the goal of the analysis.

Our suggestion is motivated by the fact that usually semicontinuous data have a complex structure: they present a clustered (spatial and/or temporal) structure; the positive values may have a highly skewed distribution; the relation between the covariates and the response may be not satisfactorily expressed by a model in which the covariates enter the distribution of the response through a linear function, and so on. In these cases, a parametric

model cannot be describe in appropriate manner the mechanism generating the data and may be opportune relax parametric assumptions to allow greater modeling flexibility. Bayesian nonparametric or semiparametric models that allow achieving this flexibility are well known in literature. However, it is unknown their use for the analysis of semicontinuous data.

As motivating example we illustrate the application of suggested semiparametric Bayesian two-part model to predict grapewine production values on a small area estimation framework. Results suggests that proposed semiparametric two-part model seems to be able to: capture the particular relationship between response variable and covariates included on the linear predictor, discriminate between the two different mixture components, handle the asymmetry of the data.

The paper is organized as follows. The suggested model is described in Section 2. Section 3 presents pseudo-real data application. Final conclusion are reported in Section 4.

## 2 The semiparametric Bayesian two-part model

To account for semicontinuity of the response variable  $Y$ , it is assumed that for each unit  $i$  ( $i = 1, \dots, n$ ) of the population  $Y_i = \delta_i Z_i$ ;  $\delta_i$  is an indicator (i.e., it takes values 0 and 1 only) independent of the continuous random variable  $Z_i$ .  $\delta_i$  indicates that  $Y_i$  came from the continuous and not from the degenerate point-mass distribution.

We define a two part model. The first part predicts  $\delta_i \mid \mathbf{w}_i$ , the second part predicts  $Y_i \mid \mathbf{x}_i$ ; where  $\mathbf{w}_i$  and  $\mathbf{x}_i$  represent two vector of explanatory variables.

In the first part, we consider a semiparametric Bernoulli regression model for data  $(\delta_i, \mathbf{w}_i')$ , where  $\delta_i$  is a binary response variable and  $\mathbf{w}_i$  an  $r$ -dimension vector of predictors (intercept included). Parametric versions of this model are characterized by the following assumption:

$$P(\delta_i = 1 \mid \mathbf{w}_i, \boldsymbol{\theta}) = E(\delta_i = 1 \mid \mathbf{w}_i, \boldsymbol{\theta}) = F_\phi \left[ t(\boldsymbol{\beta}^1, \mathbf{w}_i) \right]$$

where  $F_\phi(\cdot)$  is a distribution function on the real numbers (known up to a parameter  $\phi$ ), called the inverse link function in the context of generalized linear models, and  $t(\cdot)$  is the index function, parameterized by  $\boldsymbol{\beta}^1$ . Popular parametric versions consider a linear index function  $t(\boldsymbol{\beta}^1, \mathbf{w}_i) = \mathbf{w}_i' \boldsymbol{\beta}^1$ , and a known cumulative distribution function for  $F_\phi$ , thus allowing relatively simple treatment of the finite regression parameters,  $\boldsymbol{\theta} = \boldsymbol{\beta}^1$ . Following Jara *et al.* (2006), we consider a latent variable representation  $\delta_i = I(V_i \leq \mathbf{w}_i' \boldsymbol{\beta}^1)$  where  $V_1, \dots, V_n \sim G^1$ . We replace the parametric inverse link function  $F_\phi$  by a

general distribution  $G^1$  on which a DP prior is defined:  $G^1 \sim DP(\alpha^1 G_0^1)$ . We decided to center the prior around a logistic distribution; i.e. the baseline prior distribution  $G_0^1$  is a Logistic( $V \mid 0, 1$ ). To complete the model specification, a Gamma( $a_0^1, b_0^1$ ) for the precision parameter  $\alpha^1$  (see Escobar and West, 1995) and a Normal $_r(\beta_0^1, \mathbf{S}_{\beta_0^1})$  for regression coefficients  $\beta^1$  are given. A Metropolis-Hastings step is used to sample the full conditional distribution of the regression coefficients and precision (see Jara *et al.*, 2006).

The second part is carried on just for positive values data  $j = 1, \dots, m$ , where  $m < n$ . A DP mixture of Normal distribution (Escobar and West, 1995) for the conditional density estimation on  $Z_j \mid \mathbf{x}_j$  is used. According to Muller *et al.* (1996) we specified a DP mixture of Normals for the joint distribution of the response and predictors and we looked at the induced conditional regression. Even if, in the original paper, Muller *et al.* (1996) focussed on the mean regression function their method can be used to model the conditional density of the response giving the predictors (see Dunson *et al.*, 2007). Let  $Z_j$  and  $\mathbf{X}_j$  be the response and the  $p$  dimensional vector of continuous predictors, respectively. Further, let  $\mathbf{d}_j = (z_j; \mathbf{x}_j)'$ , with  $j = 1, \dots, m$  and  $k = p+1$  dimension. The model for the joint distribution of the response and predictors is:  $\mathbf{d}_j \sim \text{Normal}_k(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$ , with *iid* distributions for  $(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$ ,  $j = 1, \dots, m$ . For each  $j$ ,  $(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j) \sim G^2$  and  $G^2 \sim DP(\alpha^2 G_0^2)$ . The prior for the baseline distribution  $G_0^2$  is the conjugate Normal - Inverted Wishart distribution

$$G_0^2 \equiv \text{Normal}_k(\mathbf{m}_1, k_0^{-1} \boldsymbol{\Sigma}) \quad \text{Inverse Wishart}_k(\nu_1, \boldsymbol{\Psi}_1)$$

The model specification is completed when the independent priors are given:  $\alpha^2 \sim \text{Gamma}(a_0^2, b_0^2)$ ,  $\mathbf{m}_1 \sim \text{Normal}(\mathbf{m}_2, \mathbf{S}_2)$ ,  $k_0 \sim \text{Gamma}(\tau_1/2, \tau_2/2)$  and  $\boldsymbol{\Psi}_1 \sim \text{Inverse Wishart}_k(\nu_2, \boldsymbol{\Psi}_2)$ .

This second part of the model defines a weight dependent mixture models:

$$f(z) = \sum_{l=1}^{\infty} \omega_l(\mathbf{x}) \text{Normal}(\beta_{0l}^2 + \mathbf{x}' \boldsymbol{\beta}_l^2, \sigma_l^2)$$

where

$$\omega_l(\mathbf{x}) = \frac{\omega_l \text{Normal}_p(\boldsymbol{\mu}_{2l}, \boldsymbol{\Sigma}_{22l})}{\sum_{q=1}^{\infty} \omega_q \text{Normal}_p(\boldsymbol{\mu}_{2q}, \boldsymbol{\Sigma}_{22q})}$$

with  $\beta_{0l}^2 = \mu_{1l} - \boldsymbol{\Sigma}_{12l} \boldsymbol{\Sigma}_{22l}^{-1} \mu_{2l}$ ,  $\boldsymbol{\beta}_l^2 = \boldsymbol{\Sigma}_{12l} \boldsymbol{\Sigma}_{22l}^{-1}$  and  $\sigma_l^2 = \sigma_{11l}^2 - \boldsymbol{\Sigma}_{12l} \boldsymbol{\Sigma}_{22l}^{-1} \boldsymbol{\Sigma}_{21l}$ . The weights  $\omega_l$  follow a DP stick-breaking construction and the other components derive from the standard partition of the vectors of means and variance and covariance matrices given by  $\boldsymbol{\mu}_l = (\mu_{1l}, \boldsymbol{\mu}_{2l})'$  and  $\boldsymbol{\Sigma}_l = \begin{pmatrix} \sigma_{11l}^2 & \boldsymbol{\Sigma}_{12l} \\ \boldsymbol{\Sigma}_{21l} & \boldsymbol{\Sigma}_{22l} \end{pmatrix}$ .

To complete the model, we assigned the values to hyperparameters. In the following application, for example, we considered  $a_0^1 = 2$ ,  $b_0^1 = 1$ ,  $\beta_0^1 = \mathbf{0}$ ,  $\mathbf{S}_{\beta_0^1} = \text{diag}_r(10000)$ ,  $a_0^2 = 10$ ,  $b_0^2 = 1$ ,  $\nu_1 = \nu_2 = 4$ ,  $\mathbf{m}_2 = (\bar{z}, \bar{x})'$ ,  $\tau_1 = 6.01$ ,  $\tau_2 = 3.01$  and  $\mathbf{S}_2 = \Psi_2^{-1} = 0.5 \mathbf{S}$ , where  $\mathbf{S}$  is the sample variance-covariance matrix for the response and predictor.

To sum up, the two-part model parameters are the followings. The precision parameters of the DPs, respectively  $\alpha^1$  and  $\alpha^2$  for the first and the second part of the model, and the number of clusters that the DPs induce. From the first part of the model, the  $\beta^1$  regression coefficients. From the second part of the model, in the case that just one predictor is included (as on the motivating example considered): the mean  $\mathbf{m}_1$  of the Normal component of the baseline distribution  $G_0^2$  as a bivariate vector with elements  $m_{1,z}$  and  $m_{1,x}$ ; the scale matrix  $\Psi_1$  of the inverted Wishart part of the baseline distribution  $G_0^2$  as a  $2 \times 2$  symmetric matrix with elements  $\psi_{1,z}$ ,  $\psi_{1,x}$  (on the diagonal) and  $\psi_{1,zx}$  (out diagonal). Finally, the scale parameter  $k_0$  of the Normal part of the baseline distribution  $G_0^2$ .

From the first part of the model, we obtained  $E(\delta_i = 1 \mid \mathbf{w}_i)$ . From the second part of the model, we obtained in straight way the predictive distribution  $f(z_i \mid \mathbf{x}_i)$ . Finally, in order to obtain the predictive distribution  $f(y_i \mid \mathbf{x}_i, \mathbf{w}_i)$ , according to the parametric semicontinuous two-part models standard practice, we considered the product  $f(z_i \mid \mathbf{x}_i) E(\delta_i = 1 \mid \mathbf{w}_i)$ .

### 3 Motivating example: the pseudo-real data on grapewine production in Tuscany

In this section, we present an empirical evaluation of the proposed modeling by analyzing some pseudo-real agricultural data on grapewine production in Tuscany. A specific crop production is a typical case of semicontinuous output variable and the grapewine production in Tuscany does not contradict this assertion.

Our data come from two survey conducted by the Italian Statistical Institute (ISTAT): the Fifth Agricultural Census performed in 2000 (hereafter census2000) and the Farm Structure Survey performed in 2003 (hereafter FSS2003). Both census2000 and FSS2003 are a reiteration of two surveys routinely conducted by ISTAT, ten-yearly and two-yearly respectively, in order to monitor trends and transitions in the structure of farms, but also to model the impact of external developments or policy proposals. For both the census and the sample survey the unit of observation is the farm for which surface areas (measured in hectares) allocated to different crops, as well as

many other socioeconomic variables, are recorded. Two other important features of our data worthy of mention are the following: in FSS2003, as in all reiteration of this survey up until 2005, the production of each crop (quantity in quintals) has been observed and in census2000, for the first time, spatial information was collected. It consists of the geographical coordinates of each farm's administrative center.

Notwithstanding the FSS2003 uses a sample of more than 50,000 farms on the whole Italy, it is conceived to provide reliable estimates at regional level only. However, for each region it is often required to produce estimates even at sub-regional level, at least for its main crops. Direct estimates are not well-suited because only a small sample is available. Hence, we must refer to indirect estimators, which exploit the available variables collected at the census2000 as auxiliary variables. In fact, the lag time between the response and the auxiliary variables can be assumed to be negligible, because of the high correlations, among the auxiliary variables measured for the sampled farms in both the years 2000 and 2003. Nevertheless, it is obviously possible that some farms have been changed their activity.

Here we consider only the FSS2003 part for the Tuscany region; hence 2450 farms. A large number of these farms (1489) do not produce grapewine, while only a few (961) produce the majority of the total production in Tuscany and the distribution of the positive grapewine production in these farms is highly skewed. Figure 1 shows the semicontinuous nature of the grapewine production variable.

In this paper, we focused on the ability of the proposed model to predict grapewine production using some auxiliary variables; this for small area estimation end. In a small area estimation setting, once we have obtained the predictive distribution, we can extract a prediction for the out of sample units and finally obtain, using a plug-in estimator (that combines predicted and observed values), the area mean or total estimates (for different area levels).

In order to fit the model we decided to use only a part of the whole dataset of 2450 units. Hence, we randomly split the FSS2003 sample into two parts; these contain respectively 816 and 1634 farms ( $1/3$  and  $2/3$  of data). We used information on the 816 farms to predict the grapewine production for the others 1634 farms (for which the production from FSS2003 survey is registered, hence known). In this way comparison between the true values and the predicted ones from the suggested model is feasible. In the following we denoted as 'observed' the grapewine production from FSS2003 for the 816 farms, and 'true' and 'predicted' respectively the grapewine registered from FSS2003 and predicted from the suggested model for the others 1634 farms.

The selection of the covariates to be included in each of the two parts of

the model, among several socioeconomic variables available at census2000, was first performed using an explorative analysis. We conducted a preliminary parametric analysis on the data. A Logistic model has been first fitted to these data and the choice between alternative models (including different covariates) have been made basing on AIC (Akaike Information Criteria). For the first part of the model, four auxiliary variables are considered: presence/absence of surface allocated to grape wine, a relative measure of the latter on the overall cultivated surface, to be or not a grapewine seller, the slope of the farm's ground. Because in census2000 also the geographical coordinates have been registered, we easily obtained this latter covariate by merging slope information on a grid of geographical coordinates. We decided to include just one covariate in the second part of the model for a simpler analysis of the results. The 'more explicative' covariate is the surface allocated to grape wine.

For the estimation of the models, via MCMC simulation methods, we used the `DPbinary` and `DPcdensity` functions from the library `DPpackage` of the `R` package (see Jara *et al.*, 2011). A sensitivity analysis on the hyperparameters values choice has been carried out. Convergence has been checked by Gelman and Rubin (1992) convergence diagnostic criterion. The algorithm seems to converge after a few thousand iterations. However, given the very high number of (non monitored) parameters in the model, we decided to discard the first 200,000 iterations (burn-in) and to store 2000 samples (one each 100) of the following 200,000 iterations.

From the first part of the model, we obtained  $E(\delta_i = 1 \mid \mathbf{w})$ , hence the probability to have positive grapewine production for each  $i$ . The sum, over the farms, is 332.6; this suggest good performance of the first part of the model because 333 are the farms with positive grapewine production (483 with zero production). Moreover, on the 816 units modelled, the mean for units with zero production is 0.16, whereas for those with positive production is 0.76. To evaluate predictive capabilities of the first part of the model, we considered a cut-off of 0.5 over  $E(\delta_i = 1 \mid \mathbf{w})$ ; this to the end of classify (by prediction) the farms with zero or positive production. On the whole 816 units, the farms that have zero production and are rightly classified are the 51% whereas those wrongly classified are the 8%. Moreover, the farms that have positive production and are correctly classify are the 38%, but those uncorrected classified are the 3%. To sum up, according to a cut-off of 0.5, the 89% seem to be rightly classified and 11% are misclassified.

Figure 2 describes the nonparametric link function estimated from the first part of the model. It seems to be less steep for large values of the predictor  $\beta \mathbf{w}_i'$  respect to the prior distribution (i.e. the logistic).

Table 1 reports some descriptive values of the estimated posterior distri-



bution for the parameters of the semiparametric Bayesian two-part model.

Figure 3 shows estimated conditional predictive distributions  $f(y_i | x_i, \mathbf{w}_i)$  for selected values of the covariate  $x_i$  (surface allocated to grapewine from census2000) from the 1634 farms; we decided to have a description for units with  $x_i = 0, 21.48, 42.85, 124.08$ . The gray triangle represents the true grapewine production value for the farm with surface allocated to grapewine  $x_i$ . The proposed semiparametric two-part model seems to have good prediction performances. Figure 4 describes the prediction for the 1634 farms: the fitted regression function  $E(y_i | x_i, \mathbf{w}_i)$  obtained from the suggested model. We noted a particular non-linear relation between surface allocated to grapewine production from census2000 and grapewine production from FSS2003. Differences between prediction and true values are high when not support from data is given to estimate predictive distributions. Note that we obtained the predictive distribution for the semicontinuous variable by multiply a predictive distribution (obtained from the second part of the model) and an expected value (obtained from the first part). As a consequence, the 95% credibility intervals, defined for  $f(y_i | x_i, \mathbf{w}_i)$ , take into account only for the variability on estimates from the second part of the model.

## 4 Conclusions

The results, obtained from the application to data on grapewine production, show that the proposed methodology provides a reasonable and useful alternative to existing methods when assumptions of the parametric model are not valid. Moreover, an appealing feature of the suggested model is to work directly on the conditional predictive distributions. Despite the fact that the proposed methodology provides encouraging results, further research is necessary. To start with, thanks to the clustering properties of the DP mixture, the inclusion of a correlation structure could be overcome; but anyway we can provide the explicit inclusion of some (time or space) correlation structures. Moreover, regarding the 95% credibility intervals, since we have not take into account for the variability of estimates from the first part of the model, we can extend the model to cope with this variability. Finally, we can argue that because we use a mixture model on the second part, we can include on it also the degenerate component, so to consider a ‘single-part’ hierarchical nonparametric Bayesian model.



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## References

- Albert, P.S., and Shen, J. (2005). Modelling longitudinal semicontinuous emesis volume data with serial correlation in an acupuncture clinical trial. *Journal of the Royal Statistical Society: Series C* **54**, 707–720.
- Berk, K.N., and Lachenbruch, P.A. (2002). Repeated measures with zeros. *Statistical Methods in Medical Research* **11**, 303–316.
- Chandra, H., Chambers, R. (2014). Small area estimation for semicontinuous data. *Biometrical Journal*, article first published online : 24 june 2014, DOI: 10.1002/bimj.201300233
- Dreassi, E., Petrucci, A., and Rocco, E. (2014). Small area estimation for semicontinuous skewed spatial data: an application to the grapewine production in Tuscany. *Biometrical Journal* **56**, 141–156.
- Duan, N., Manning, W.G., Morris, C. N., and Newhouse, J.P. (1983). A comparison of alternative models for the demand for medical care. *Journal of Economic and Business Statistics* **1**, 115–126.
- Dunson, D.B., Pillai, N., and Park, J.H. (2007). Bayesian density regression. *Journal of the Royal Statistical Society B* **69**, 163–183.
- Escobar, M.D. (1994). Estimating normal means with a Dirichlet process prior. *Journal of the American Statistical Association* **89**, 268–277.
- Escobar, M.D., and West, M. (1995). Bayesian density estimation and inference using mixtures. *Journal of the American Statistical Association* **90**, 577–588.
- Gelman, A., and Rubin, D.R. (1992). Inference from iterative simulation using multiple sequences (with discussion). *Statistical Science* **7**, 457–511.
- Gosh, P., and Albert, P.S. (2009) A Bayesian analysis for longitudinal semicontinuous data with an application to an acupuncture clinical trial. *Computational Statistics and Data Analysis* **53**, 699–706.

- Jara, A., Garcia-Zatera, M.J., Lesaffre, E. (2006). Semiparametric Bayesian Analysis of Misclassified Binary Data. *XXIII International Biometric Conference*, July 16-21, Montréal, Canada.
- Jara, A., Hanson, T.E., Quintana F.A., Muller, P., and Rosner, G.L. (2011). DPpackage: Bayesian Semi and Nonparametric Modeling in R. *Journal of Statistical Software* **40**, 1–30.
- Muller, P., Erkanli, A., and West, M. (1996). Bayesian curve fitting using multivariate normal mixtures. *Biometrika* **7**, 975–996.
- Neal, R.M. (2000). Markov Chain sampling methods for Dirichlet process mixture models. *Journal of Computational and Graphical Statistics* **9**, 249–265.
- Olsen, M.K., and Schafer, J.L. (2001). A two-part random-effects model for semicontinuous longitudinal data. *Journal of the American Statistical Association* **96**, 730–745.
- Pfeffermann, D., Terry, B., and Moura, F.A.S. (2008). Small area estimation under a two-part random effects model with application to estimation of literacy in developing countries. *Survey Methodology* **34**, 235–249.
- Tooze, J.A., Grunwald, G.K., and Jones, R.H. (2002). Analysis of repeated measures data with clumping at zero. *Statistical Methods in Medical Research* **11**, 341–355.

Table 1: Data on grapewine production: estimate of the parameters of the semiparametric Bayesian two-part model with their 95% credibility intervals (95% CI).

parameter	mean	95% CI
$\beta_0^1$ (intercept)	-2.622	-4.606 ; -1.260
$\beta_1^1$ (surface presence/absence)	3.823	2.163 ; 5.818
$\beta_2^1$ (relative grapewine surface)	1.744	0.271 ; 4.019
$\beta_3^1$ (to be seller yes/no)	0.826	0.174 ; 1.853
$\beta_4^1$ (slope)	-0.638	-1.483 ; 0.025
$\alpha^1$	16.525	7.057 ; 26.895
number of clusters from DP first part	79.194	41 ; 114
$m_{1,z}$	168.400	66.740 ; 291.900
$m_{1,x}$	3.242	0.454 ; 6.584
$k_0$	0.011	0.002 ; 0.032
$\psi_{1,z}$	0.002	0.001 ; 0.003
$\psi_{1,x}$	1.422	0.797 ; 2.424
$\psi_{1,zx}$	-0.017	-0.039 ; -0.002
$\alpha^2$	6.917	4.017 ; 10.658
number of clusters from DP second part	24.173	16 ; 33

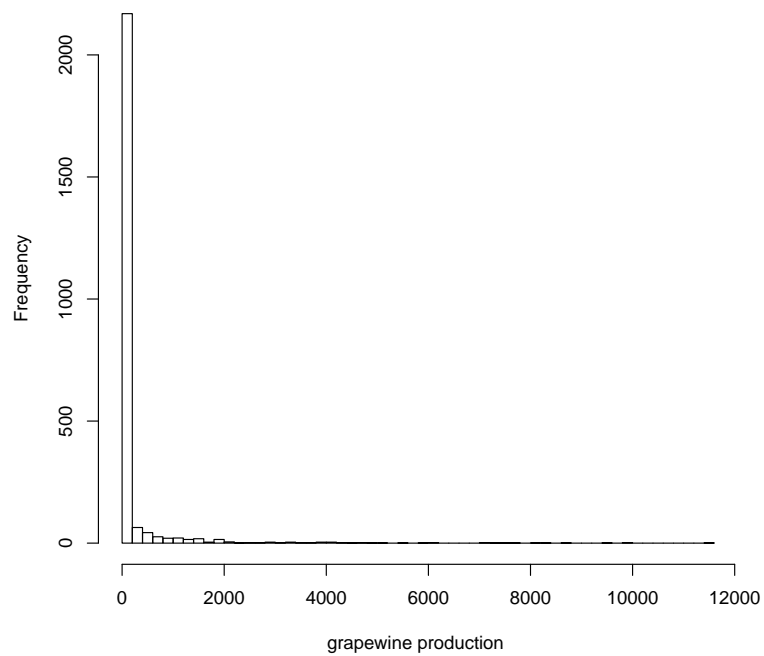


Figure 1: Data on grapewine production: the histogram for the 2450 units from the FSS2003 survey.

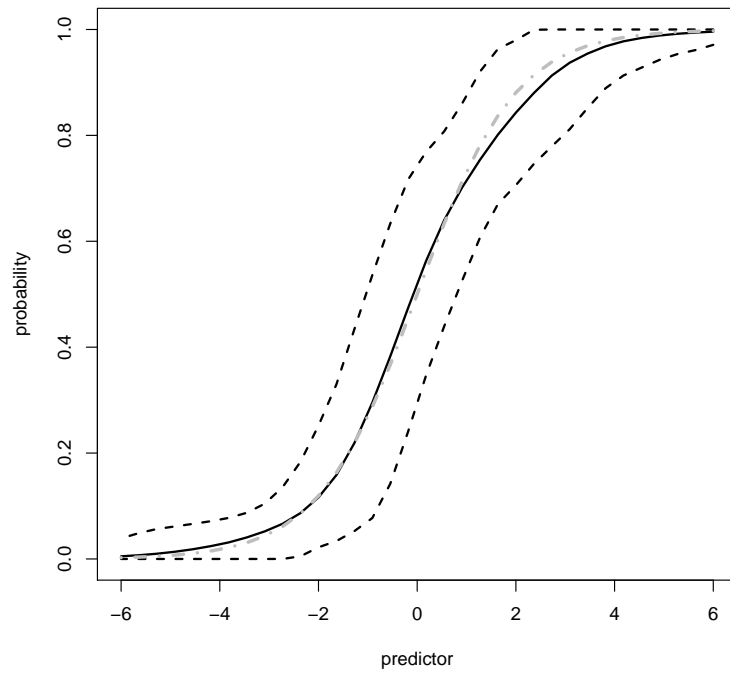


Figure 2: Data on grapewine production: estimated link function (black solid line) and its 95% credibility intervals (black dashed lines) *versus* parametric logistic link function (gray dashed line).

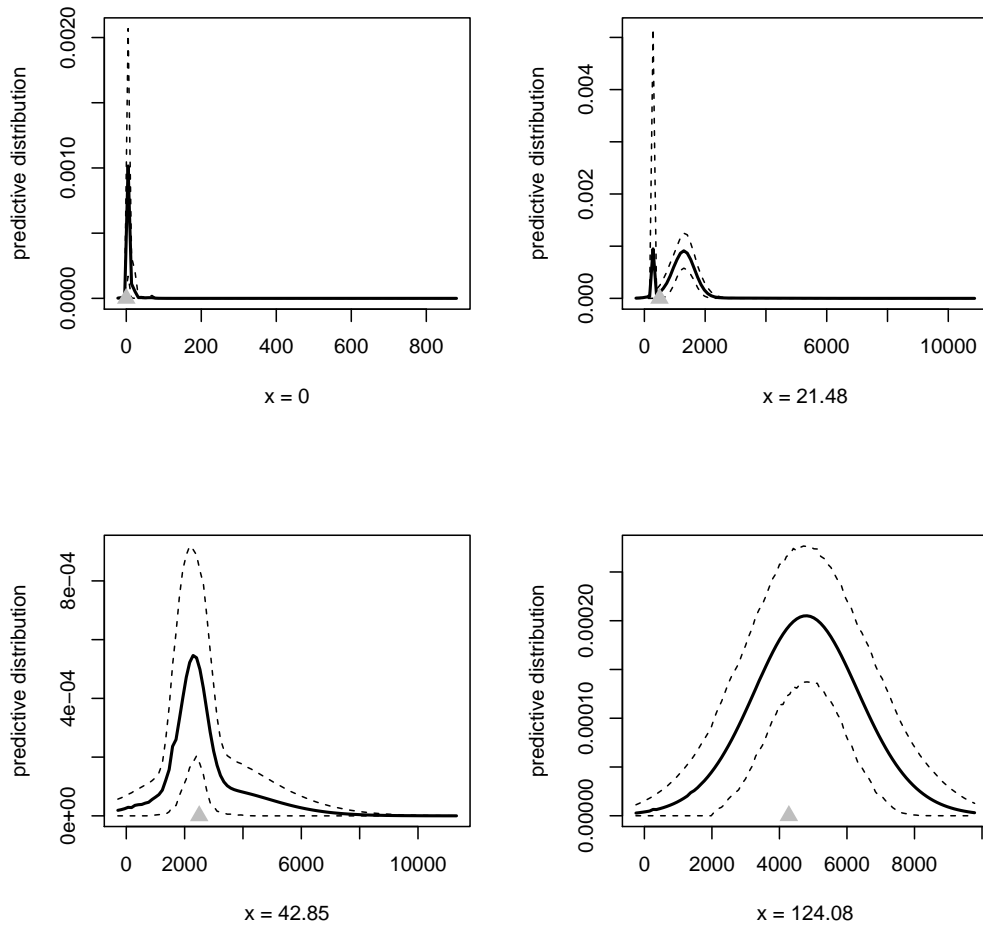


Figure 3: Data on grapevine production: conditional predictive distribution  $f(y_i | x_i, \mathbf{w}_i)$  (solid line) with its 95% credibility intervals (dashed lines); true grapevine production value (gray triangle).

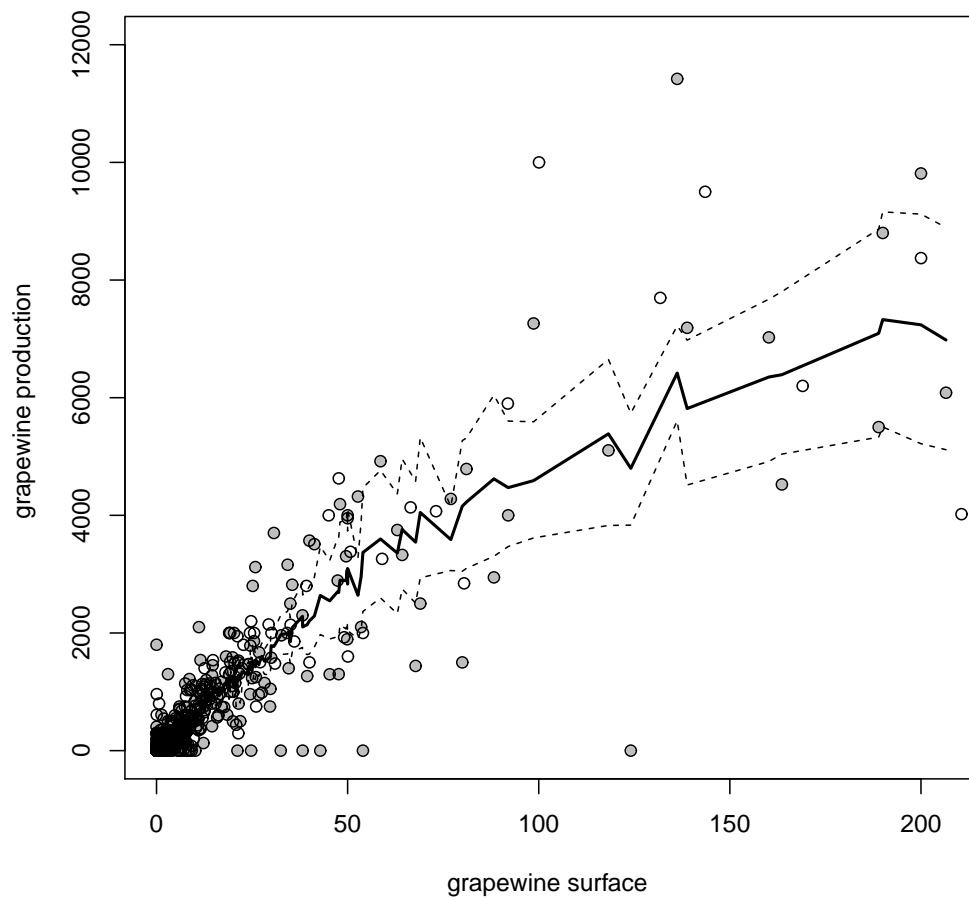


Figure 4: Data on grapewine production: (white circle) observed data; (gray circle) true value; (solid line) fitted regression prediction function with its 95% credibility intervals (dashed lines).